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**THE LIFE CYCLE OF THE FIRM WITH DEBT AND CAPITAL  
INCOME TAXES**

By Bert Brys, A. Lans Bovenberg

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# The life cycle of the firm with debt and capital income taxes<sup>◊</sup>

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## Abstract

This paper analyses the impact of capital income taxes on financial and investment decisions of corporations. Extending Sinn's (1991) nucleus theory of the firm with debt finance, the model determines the optimal sources of finance (debt, newly issued equity or retained earnings), the optimal use of the investment's earnings (dividends, retentions, interest payments or debt redemption), and the optimal capital accumulation throughout the life cycle of the firm.

Keywords: tax burden; capital income taxation; firm behaviour

JEL codes: H32, G32, D21

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## 1. Introduction

Sinn (1990, 1991a, 1991b) studies a dynamic life-cycle model of the firm. If retained earnings are a cheaper source of finance than newly issued equity, a young firm issues only a nucleus of new equity. The return on the initial investment is retained and reinvested during an internal growth phase of the firm. The firm keeps reinvesting its earnings for a while. Eventually, however, the firm stops investing and starts distributing dividends. Sinn's work establishes how the capital income tax system affects the cost of capital throughout the entire life cycle of a firm. In particular, the cost of newly issued equity may exceed the cost of new equity as derived by King and Fullerton (1984). Intuitively, the more new equity the firm issues initially, the lower is the amount of investment that the firm can finance with cheaper retained earnings. This effect adds to the opportunity cost of newly issued equity. During the initial growth phase when the firm uses all its earnings for reinvestment, the cost of retained earnings differs from that computed by King and Fullerton as well. Indeed, during this internal growth phase, all the returns on investment are retained and reinvested and not, as assumed by King and Fullerton, distributed as dividends.

Also van Schijndel (1986, 1988) studies the impact of corporate and personal taxes on the firm's financial, investment and dividend decisions in a dynamic setting. In contrast to Sinn (1991a), van Schijndel focuses on a finite horizon model and assumes that the firm owns an exogenous initial amount of equity. The firm is not allowed to issue new equity. Due to the model's finite horizon and the differential tax treatment of capital gains and dividends, the firm might find it optimal at the end of the time interval to stop distributing dividends and to retain the earnings. In contrast to Sinn, van Schijndel does allow for debt financing.

We extend the results derived by van Schijndel (1986, 1988) by introducing debt in Sinn's dynamic life-cycle model of the firm. By allowing for an infinite horizon, the mature firm distributes dividends forever. Moreover, the firm endogenously determines the optimal amount of initial equity or debt-financed capital. The firm is allowed to issue new equity and new debt at any time. These extensions allow us to study the effects of the capital income tax system during the entire life cycle of the firm. In particular, as in Sinn (1991a, 1991b), we derive the cost of capital along the entire optimal path. Moreover, we investigate the optimal sources of finance (debt, newly issued equity or retained earnings), the optimal use of the investment's earnings (dividends, retentions, interest payments or debt redemption) and the tax burden throughout the life cycle of the firm. In this way, we extend Sinn's analysis by allowing for debt finance and debt redemption. The possibility of financing initial investment with debt allows the firm to more rapidly accumulate earnings that can be distributed or retained and reinvested. This may reduce the need to issue tax-disadvantaged new equity. As the firm matures, it redeems its debt if retained earnings are tax favoured compared to debt. In this way, debt finance introduces two additional phases compared to the analysis of Sinn: one internal growth phase during which investment is financed by a combination of debt finance and retained earnings and one phase during which earnings are used to redeem debt rather to finance real investment.

Section 2 introduces the life-cycle model of the firm. The steady-state conditions determining the tax preferences for the various types of financing are investigated in section 3. Section 4 discusses the model's solution if retained earnings are the least preferred source of finance. Section 5 explores the firm's finance and investment behaviour if retained earnings are preferred to debt and if newly issued equity is the least preferred source of finance. Section 6 concludes.

## 2. The life-cycle model of the firm

As in Sinn (1991a)<sup>1</sup>, the firm produces output with capital  $K$  as sole production factor, which is assumed not to depreciate. Commodity prices are constant and normalised to unity. The firm's revenue and output are described by the production function  $f(K)$ , which satisfies  $f'(K) > 0$ ,  $f''(K) < 0$  and  $f(0) = f'(\infty) = 0$ ,  $f'(0) = \infty$ . The firm finances the investment  $I$  with newly issued equity  $Q$ , with newly issued debt  $S_f$ , or with retained earnings.  $D_f$  denotes the firm's stock of debt. The model allows for a positive corporate tax rate on distributed profits  $\tau_d$  and on retained profits  $\tau_r$ .  $\tau_p$  stands for the personal income tax rate on dividends and interest income. Capital gains are taxed on an accrual basis. The tax rate on realised capital gains is transformed into an equivalent tax rate  $\tau_c$  on accrued capital gains.  $1 - \tau_d$  is denoted by  $\theta_d$ . A similar notation applies to the  $1 - \tau_r$ ,  $1 - \tau_p$  and  $1 - \tau_c$ .

The (representative) shareholder wants the firm to maximise the initial value of the shares net of the originally injected equity. The shareholder looks through the corporate veil and perfectly foresees all variables in the model. The shareholder can lend at the exogenous interest rate  $r$ .

The market value of the shares  $M$  is implicitly determined by the arbitrage condition (1). In particular, the shareholders are indifferent between retaining shares at a value of  $M$  or exchanging these shares for bonds. This implies that the after-tax return on shares equals the potential after-tax returns  $\theta_p r M$  from holding bonds

$$\theta_p \theta_d \pi^d + \dot{m} z \theta_c + (z \dot{m} - Q) \theta_c = \theta_p r M. \quad (1)$$

The left-hand side of this expression represents the net return on shares ( $m$  is the price of a share,  $z$  is the number of outstanding shares; the dot stands for time derivative of these variables). The after-tax return consists of three components:  $\theta_p \theta_d \pi^d$  is the net dividend paid out to shareholders,  $\dot{m} z \theta_c$  stands for the capital gain from the existing stock of shares

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<sup>1</sup> The model's notation is borrowed from Sinn (1987). We employ optimal control theory to solve the model analytically. The solution procedure of van Loon (1983) is used (see Appendix C), as applied in van Schijndel (1986, 1988), van Hilten et al (1993) and Kari (1999).

net of the capital gains tax, and  $(zm - Q)\theta_c$  represents the net-of-tax capital gain from buying new shares at a price below market value.

Gross dividends  $\pi^d$  consist of the firm's revenue net of the firm's interest payments plus the attracted newly issued debt and equity net of the firm's investment and the corporate tax on retained profits:

$$\pi^d = f(K) - rD_f + S_f + Q - I - \tau_r(f(K) - rD_f - \pi^d). \quad (2)$$

The firm's taxable earnings are assumed to be large enough such that interest payments are deductible from taxable corporate earnings.

The firm's optimisation problem can be formalized as follows<sup>2</sup>:

$$\begin{aligned} \text{Max} \quad & M(t_1) - K(t_1) + D_f(t_1) \\ & \{I, S_f, Q, K(t_1), D_f(t_1)\} \end{aligned} \quad (3)$$

$$\begin{aligned} \text{Sub to:} \quad & M(t_1) = \int_{t_1}^{\infty} \left[ \frac{\theta_p \theta_d \pi^d(v)}{\theta_c} - Q(v) \right] \cdot e^{-\int_{t_1}^v \frac{\theta_p}{\theta_c} r(s) ds} dv \\ & \pi^d = f(K) - rD_f + \frac{1}{\theta_r} [S_f + Q - I] \\ & \dot{K} = I \\ & \dot{D}_f = S_f \\ & S_f \leq \alpha I \\ & D_f(t_1) \leq \alpha K(t_1) \\ & Q \geq 0 \\ & \theta_p \theta_d \pi^d \geq 0 \end{aligned}$$

The objective function represents the firm's initial period's market value of the shares  $M(t_1)$ , net of the originally injected equity. This initial equity corresponds to the first-period invested

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<sup>2</sup> Using  $\dot{M} = \dot{m}z + z\dot{m}$ , we can write (1) as  $\dot{M} = -\frac{\theta_p \theta_d}{\theta_c} \pi^d + Q + \frac{\theta_p}{\theta_c} rM$ . Integration of this differential

equation yields the value of the firm's equity  $M(t) = \int_t^{\infty} \left[ \frac{\theta_p \theta_d \pi^d(v)}{\theta_c} - Q(v) \right] \cdot e^{-\int_t^v \frac{\theta_p}{\theta_c} r(s) ds} dv$ , where we have

used the transversality condition  $\lim_{v \rightarrow \infty} \left\{ \left[ \frac{\theta_p \theta_d \pi^d(v)}{\theta_c} - Q(v) \right] \cdot e^{-\int_t^v \frac{\theta_p}{\theta_c} r(s) ds} \right\} = 0$ .

capital  $K(t_1)$  minus the first-period debt  $D_f(t_1)$ . The capital stock  $K$  and the stock of debt  $D_f$  are the state variables. The control variables are investment  $I$ , newly issued equity  $Q$  and newly issued debt  $S_f$ . At most, a share  $\alpha$  of the investment can be financed with newly issued debt, where  $\alpha$  is the maximum debt-capital ratio. The same condition holds during the initial period. Debt can thus never exceed  $(\alpha \cdot 100)\%$  of the capital stock. The amount of newly issued equity is non-negative. Hence, the firm is not allowed to repurchase shares. Moreover, negative after-tax dividends are excluded.

### 3. Preferred sources of finance

This section investigates how the capital income tax system impacts the firm's steady-state preferences with respect to the three sources of finance: debt, newly issued equity or retained earnings. The derivations can be found in Appendix A.1.

The firm prefers retained earnings ( $RE$ ) over newly issued equity ( $NE$ ), if capital gains are taxed less heavily than dividends are:

$$\begin{array}{ccc} > & & > \\ NE \approx RE & \Leftrightarrow & \theta_p \theta_d = \theta_c \theta_r . \\ < & & < \end{array} \quad (4)$$

The firm prefers to reinvest one euro of before-tax earnings (which yields  $\theta_c \theta_r$  after taxes to the shareholder) if reinvestment is more profitable than distributing the earnings as dividends (which yields  $\theta_p \theta_d$  after taxes to the shareholder).

We can compare retained earnings also with newly issued debt as a source of financing. In contrast to dividends, interest payments are deductible from taxable corporate earnings. Hence, debt is taxed only once, namely at the personal level. The firm then prefers retained earnings ( $RE$ ) to newly issued debt ( $DF$ ), if the taxation of retained earnings at both the corporate level (reflected in  $\tau_r$ ) and personal level (reflected in  $\tau_c$ ) is less than the personal taxation on interest payments:

$$\begin{array}{ccc} > & & > \\ DF \approx RE & \Leftrightarrow & \theta_p = \theta_c \theta_r . \\ < & & < \end{array} \quad (5)$$

Similarly, the firm in the steady state prefers newly issued debt ( $DF$ ) to newly issued equity ( $NE$ ), if interest payments are taxed less heavily than dividends are:

$$\begin{array}{ccc} > & & > \\ DF \approx NE & \Leftrightarrow & \theta_p = \theta_p \theta_d . \\ < & & < \end{array} \quad (6)$$

#### 4. The model's solution if $\theta_p \geq \theta_p \theta_d \geq \theta_c \theta_r$

This section assumes that  $\theta_p \geq \theta_p \theta_d \geq \theta_c \theta_r$ . These inequities imply that the firm faces incentives to distribute its before-tax earnings and to finance additional investment with external sources of finance. Moreover, debt is preferred to newly issued equity. Consequently, the firm immediately attracts the optimal amount of debt and equity-financed capital and starts distributing dividends. The firm invests until the marginal increase in value of the firm's equity  $q_K$  as the result of a unit increase in the capital stock equals the cost of an additional unit of investment (see Appendix B)

$$q_K = 1 - \tau_d \alpha . \quad (7)$$

The firm prefers to finance an additional investment of one euro entirely with debt, which costs only  $\theta_d$  euro in the steady state. However, the firm is allowed to finance only  $(\alpha \cdot 100)\%$  of the investment with debt. Consequently, the firm will have to finance the remaining  $((1 - \alpha) \cdot 100)\%$  with newly issued equity at unit cost. The cost of the marginal investment is then a weighted-average of the costs of debt and newly issued equity.

The investment's minimum required before-tax return (cost of capital)  $f'(K)$  amounts to

$$f'(K) = \alpha r + (1 - \alpha) \frac{r}{\theta_d} . \quad (8)$$

The marginal investment financed with newly issued equity yields a return  $f'(K)$ , which is distributed as dividends. This return is taxed under both the corporate and personal income tax. Consequently, the household's after-tax income equals  $\theta_p \theta_d f'(K)$ . This investment has to yield a return equal to the household's opportunity return  $\theta_p r$ . The cost of capital on investment financed with newly issued equity then amounts to  $f'(K) = \frac{r}{\theta_d}$ . Similarly, a marginal debt-financed investment yields a return  $f'(K)$ , which is taxed under the income tax. The household's after-tax interest payments  $\theta_p f'(K)$  have to be equal to the opportunity return  $\theta_p r$ . The cost of capital on debt-financed investment then is given by  $f'(K) = r$ . The weighted-average cost of capital (8) then follows.

#### 5. The model's solution if $\theta_c \theta_r > \theta_p > \theta_p \theta_d$

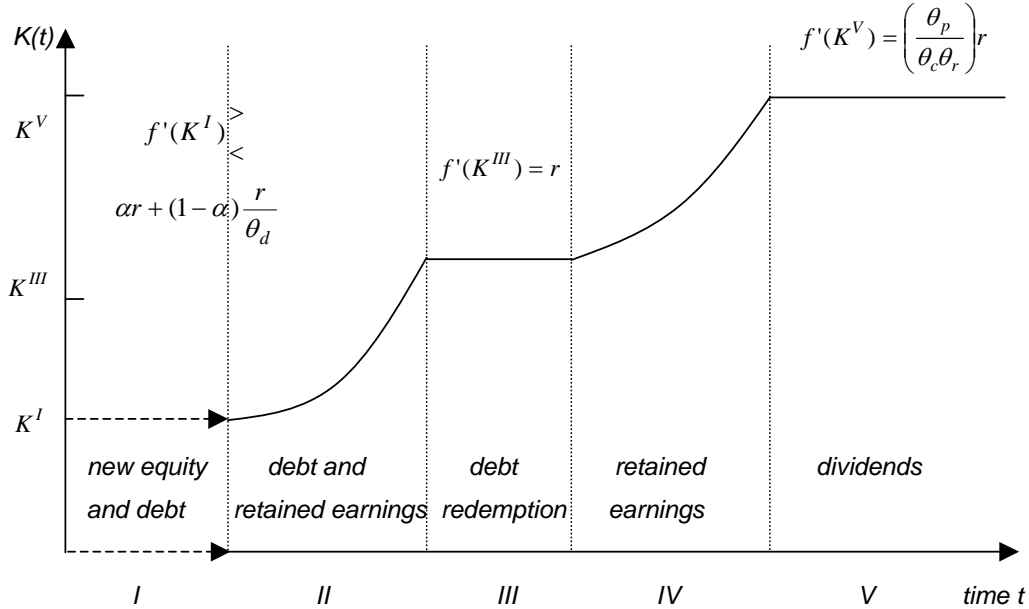
If  $\theta_c \theta_r > \theta_p > \theta_p \theta_d$ , the firm prefers retained earnings to debt and newly issued equity as a source of finance in the steady state. In order to defer the taxes on distributed profits  $\tau_d + \tau_p (1 - \tau_d)$ , the firm prefers to retain and reinvest the profits instead of distributing them and then financing additional investment with newly issued equity or debt. If external sources



of finance must be attracted, the firm prefers debt to newly issued equity. The solution of the problem consists of five successive phases. Appendix C contains the formal derivations.

The capital stock along the optimal path over time is presented in *figure 1*.

*figure 1*



Since a newly founded firm does not yet possess retained earnings to finance investment, the firm has to attract external sources of finance during phase I. The firm faces an incentive to finance the initial investment with debt until the cost of capital  $f'(K)$  equals the interest rate  $r$ . The firm, however, also has to issue new equity because the limits on the debt-equity ratio imply that the firm can finance only part of the investment with debt. The firm issues only a nucleus of new equity in order to take advantage of the cheaper retained earnings as source of finance. Section 5.1 demonstrates that the initial capital stock satisfies

$f'(K(t_1)) > \alpha r + (1-\alpha) \frac{r}{\theta_d}$  if the production share of capital in a Cobb-Douglas production

function is sufficiently small. During phase II (section 5.2), the firm finances additional investment with debt and retained earnings until  $f'(K) = r$ . At this point, it is no longer optimal to finance additional investment with debt. Subsequently, the firm redeems its entire debt (phase III, section 5.3). In order to defer the taxes on distributed dividends, the firm continues to finance the investment with retained earnings, even though the return on the investment is lower than the interest rate (phase IV, section 5.4). The shareholder enjoys a higher return if the firm retains and reinvests its earnings compared to the firm distributing its earnings as dividends and the household investing the resulting after-tax dividends in bonds.

The firm invests until the return on the investment is reduced to  $f'(K) = \frac{\theta_p}{\theta_c \theta_r} r$ . During

phase V (section 5.5), the firm stops investing and starts distributing dividends because the shareholder prefers to invest the after-tax dividends in bonds. Section 5.6 discusses the optimal path over time.

### 5.1 Phase I: starting the firm

The initial capital stock is financed partly with debt and partly with newly issued equity because the newly founded firm does not yet possess retained earnings and can finance only a part  $\alpha$  of the investment with debt:  $D_f(t_1) = \alpha K(t_1)$ .

#### 5.1.1 The marginal value of capital

The firm invests until the marginal benefit, which is the marginal increase in value of the firm's equity  $q_K(t_1)$ , equals the marginal cost of the investment (see Appendix D.1). A part  $1 - \alpha$  of the marginal investment is financed with newly issued equity at unit cost. The cost of debt, which accounts for an investment share  $\alpha$ , exceeds the steady-state value. An additional unit of debt implies additional interest payments, which not only reduce the earnings that can be distributed as dividends, but also decrease the funds available for investment. The foregone gain of the additional investment should be added to the net present value of the reduction in distributed dividends. Hence, the cost of an additional unit of debt exceeds  $1 - \tau_d$  (see Appendix D.1). Consequently, the cost of the marginal investment exceeds  $(1 - \alpha) + \alpha(1 - \tau_d) = 1 - \alpha\tau_d$ . The optimal initial investment therefore satisfies

$$q_K(t_1) > 1 - \alpha\tau_d . \quad (9)$$

#### 5.1.2 The cost of capital if $\alpha = 0$

This sub-section studies the cost of capital of the initial investment that can be financed only with newly issued equity  $\alpha = 0$ . If dividends are taxed less than capital gains (see section 4), the firm issues new equity until  $f'(K) = \frac{r}{\theta_d}$ . If capital gains are taxed less than dividends,

this result changes as a result of two effects, which work in opposite directions. As a result of the first effect (the 'internal versus external equity-cost' effect), the firm faces an incentive to issue initially less new equity in order to finance investment with the 'cheaper' retained earnings. According to the second effect (the 'time to maturity' effect), in contrast, the firm issues initially more new equity in order to shorten the firm's internal growth phase. The remainder of this section discusses these two effects and introduces a condition that determines when the 'internal versus external equity-cost' effect dominates the 'time to maturity' effect.

#### The 'internal versus external equity-cost' effect

Because  $\theta_c\theta_r > \theta_p\theta_d$ , the firm prefers retained earnings to newly issued equity as a source of finance. Instead of financing investment directly with newly issued equity at unit cost, the firm thus faces an incentive to postpone the investment until it generates sufficient earnings

that can be retained. For instance, the firm might wait to invest until it has  $\left(\frac{1}{\theta_c \theta_r}\right)$  of before-tax earnings. The additional unit of investment financed with retained earnings costs the household only  $\left(\frac{\theta_p \theta_d}{\theta_c \theta_r}\right)$  in terms of foregone dividends, which is lower than the unit cost of the investment financed immediately with newly issued equity. As pointed out by Sinn (1991a), this preferential tax treatment of retained earnings as a source of finance raises the opportunity cost of initially issued new equity. An additional unit of investment financed with newly issued equity implies that the firm foregoes the opportunity to employ instead the cheaper retained earnings as a source of finance. This opportunity cost must be added to the cost of capital in case of no-deferral  $\frac{r}{\theta_d}$  (section 4).

#### *The 'time to maturity' effect*

If the firm issues initially less new equity, however, it takes more time to obtain a substantial amount of retained earnings. The firm then requires more time to become mature and to start distributing dividends. Consequently, the value of the firm's equity, which equals the present value of the after-tax dividends, declines if the firm issues initially too little new equity. This second effect, which originates in the time value of money, explains not only why the firm does not issue an infinitesimally small amount of initial new equity, but also why the firm might actually issue a substantial amount of initial new equity such that the marginal return on the investment is lower than  $\frac{r}{\theta_d}$ . Indeed, the gain of receiving dividends earlier is an opportunity return of newly issued equity, which might reduce the cost of capital below the conventional value  $\frac{r}{\theta_d}$ .

#### *General condition*

The pure profit implied by the concave production function  $f(K)$  can be interpreted as the return to a hidden fixed factor of production. Output is assumed to be linearly homogeneous in capital and the hidden factor of production<sup>3</sup>.  $\beta$  is the production elasticity of capital and  $\sigma$  is the Hicksian elasticity of substitution between capital and the hidden production factor. A

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<sup>3</sup> Assume that the firm's production function  $F(\cdot)$  is linearly homogeneous in its two arguments, capital  $\tilde{K}$  and another production factor  $N$ . The assumption of linear homogeneity allows working with the production function in intensive form. Consequently,  $F(\frac{\tilde{K}}{N}, 1) = \frac{1}{N} F(\tilde{K}, N)$ . Moreover,  $K = \frac{\tilde{K}}{N}$  and  $f(K) = F(\frac{\tilde{K}}{N}, 1)$ . Therefore, output/revenue per unit of hidden factor is written as a function of capital per unit of hidden production factor.

sufficient condition for the cost of capital of investment financed with newly issued equity to be higher than  $\frac{r}{\theta_d}$  is  $\left[1 - \frac{\theta_p \theta_d}{\theta_c \theta_r}\right] < \frac{(1-\beta)/\beta}{\sigma}$  (see Appendix D.2).

#### Condition on $\beta$

This condition can be expressed as a requirement only on the production share of capital  $\beta$  if output is described by a constant-returns-to-scale Cobb-Douglas production function, which implies that  $\sigma = 1$ . The ‘internal versus external equity-cost’ effect dominates the ‘time to maturity’ effect, which implies a value of the cost of capital higher than  $\frac{r}{\theta_d}$ , if  $\beta$  satisfies

$$f'(K(t_1)) > \frac{r}{\theta_d} \quad \Leftrightarrow \quad \beta < \frac{1}{\left[1 - \frac{\theta_p \theta_d}{\theta_c \theta_r}\right] + 1} . \quad (10)$$

Appendix E. demonstrates that the length of the internal growth phase, which determines the importance of the ‘time to maturity’ effect, is increasing in  $\beta$ . For a given difference in investment costs  $\left(1 - \frac{\theta_p \theta_d}{\theta_c \theta_r}\right)$ , the length of the internal growth phase should not be too large in order for the ‘internal versus external equity-cost’ effect to dominate the ‘time to maturity’ effect. In particular, if  $\beta < \frac{1}{\left[1 - \frac{\theta_p \theta_d}{\theta_c \theta_r}\right] + 1}$ , the effect of the difference in investment costs is

larger than the effect of the time needed to reach maturity. As a direct consequence, the cost of capital on investment financed with newly issued equity then satisfies  $f'(K(t_1)) > \frac{r}{\theta_d}$ .

#### 5.1.3 The cost of capital if $\alpha = 1$

If  $\alpha = 1$ , the cost of capital on a marginal investment financed entirely with debt satisfies (see Appendix D.2)

$$f'(K(t_1)) = r . \quad (11)$$

As opposed to newly issued equity, debt does not compete with retained earnings. Even though retained earnings are the most preferred source of finance, the firm will issue debt as long as the investment’s return exceeds the interest rate. The firm does not want to wait to invest until it possesses retained earnings. In fact, once the firm generates earnings, they can be used to redeem the debt. In terms of foregone dividends, it makes no difference whether the firm employs the earnings for investment or redemption of the firm’s debt. Moreover, both strategies imply that the firm will be entirely equity-financed once the debt is

redeemed. The firm thus faces an incentive to issue new debt as long as the investment's return exceeds or equals the interest rate. Indeed, immediate debt-financed investment allows the firm to reach the stage of maturity earlier.

#### 5.1.4 The cost of capital if $0 \leq \alpha < 1$

Because  $(\alpha \cdot 100)\%$  of the marginal investment is financed with debt and  $((1-\alpha) \cdot 100)\%$  is financed with newly issued equity, the cost of capital during the initial period satisfies (see Appendix D.2)

$$f'(K(t_1)) > \alpha r + (1-\alpha) \frac{r}{\theta_d} > r \quad \Leftrightarrow \quad \beta < \frac{1}{\left[1 - \frac{\theta_p \theta_d}{\theta_c \theta_r}\right] + 1} . \quad (12)$$

## 5.2 Phase II: internal growth phase (debt and retained earnings)

During phase II, the firm finances additional investment with debt and retained earnings. The firm continues to issue debt because the before-tax return on investment exceeds the interest rate. In view of the constraint  $S_f = \alpha I$ , the firm attracts  $\frac{\alpha}{(1-\alpha)}$  units of debt for every

unit of retained earnings. With the firm generating  $\theta_r(f(K) - rD_f)$  units of retained earnings, it can set investment  $I$  equal to  $\frac{\theta_r(f(K) - rD_f)}{(1-\alpha)}$ .

### 5.2.1 Cost of capital if $\alpha = 0$

Each unit of retained and reinvested before-tax profits must yield the household's opportunity return  $\theta_p r$ . If the firm reinvests the retained profits and the return on this investment is

retained as well, the household realises the return  $\theta_c \left[ \theta_r f'(K) + \frac{\dot{q}_K}{q_K} \right]$ , where  $q_K$  denotes the

marginal value of an additional unit of capital. The direct return on the investment is taxed at

the level of the firm at the rate  $\tau_r$ . The household realises also a capital loss ( $\frac{\dot{q}_K}{q_K}$  is

negative) because decreasing returns in investment implies that an investment financed with the retention of the return on a prior investment yields a return lower than the originally retained return. The overall change in the firm's market value is then taxed under the capital gains tax. Because only  $\theta_c \theta_r$  euro of after-tax profits are invested, the household's total

return amounts to  $\theta_c \theta_r \cdot 1 \cdot \theta_c \left[ \theta_r f'(K) + \frac{\dot{q}_K}{q_K} \right]$ . Given the household's required opportunity

return  $\theta_c \theta_r r \theta_p$ , the cost of capital amounts to

$$f'(K) = \frac{\theta_p}{\theta_c \theta_r} r - \frac{\dot{q}_K / q_K}{\theta_r} . \quad (13)$$

### 5.2.2 Cost of capital if $0 \leq \alpha < 1$

The firm faces an incentive to issue debt because the before-tax return on the investment exceeds the interest rate. Put differently, the gain of an additional unit of (debt-financed) capital  $q_K$  exceeds the cost of an additional unit of debt  $-q_D$ , which yields an additional return  $\frac{q_K + q_D}{q_K} > 0$  for debt-financed investment. This additional gain ceteris paribus decreases the investment's required return (see Appendix C.2)

$$f'(K) = \left[ \frac{1}{1 + \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{q_K + q_D}{q_K} \right)} \right] \left[ \frac{\theta_p}{\theta_c \theta_r} r - \frac{\dot{q}_K / q_K}{\theta_r} \right] > r . \quad (14)$$

### 5.3 Phase III: debt redemption

During phase III, the firm neither invests nor distributes dividends. The firm uses its earnings to redeem its debt  $S_f = -\theta_r (f(K) - rD_f)$  until  $D_f = 0$  (see Appendix C.2). This will take some time. The firm stops issuing new debt because the investment's return equals the interest rate  $f'(K) = r$ . The increase in value of the firm's equity as a result of a unit increase in the capital stock  $q_K$  therefore equals the decrease in value of the firm's equity as a result of an additional unit of debt  $-q_D$ .

### 5.4 Phase IV: internal growth phase (retained earnings)

During phase IV, the firm continues to invest. The investment  $I = \theta_r f(K)$  is financed only with retained earnings. The profits of the investment are retained and reinvested. The cost of capital during this internal growth phase amounts to (see also section 5.2 and Appendix C.2)

$$f'(K) = \frac{\theta_p}{\theta_c \theta_r} r - \frac{\dot{q}_K / q_K}{\theta_r} > \frac{\theta_p}{\theta_c \theta_r} r . \quad (15)$$

In order to defer the taxes on distributed dividends and because capital gains are taxed less heavily than interest payments, the firm finances additional investment with retained earnings, even though the return on the investment is lower than the interest rate. The shareholder realises a higher return if the firm retains and reinvests the earnings than if the firm distributes the earnings as dividends and the household invests the after-tax dividends in bonds.

### 5.5 Phase V: distribution of dividends

During phase V, the firm no longer invests and distributes all profits as dividends so that  $\pi^d = f(K)$ . The firm invests until the marginal increase in value of the equity  $q_K$  equals the cost of the additional investment, which are the net dividends foregone (see Appendix C.2):

$$q_K = \frac{\theta_p \theta_d}{\theta_c \theta_r} < 1. \quad (16)$$

If the firm retains and reinvests an additional euro, the after-tax increase in value of the firm's equity equals  $\theta_c \theta_r q_K$ . If the firm distributes the additional euro, the household receives after-tax dividends  $\theta_p \theta_d$ . The firm invests until the household is indifferent between retaining and distributing the firm's earnings. Since capital gains are taxed at lower rates than distributed dividends, the marginal increase in value of the equity is thus lower than one euro.

The household is indifferent between an investment financed with retained earnings and distributing the earnings and investing the proceeds in market debt. If the firm retains one euro of before-tax profits, it can reinvest  $\theta_c \theta_r$  of after-tax retained earnings. This investment yields a return  $f'(K)$ , which is distributed as dividends. Consequently, the household realises an after-tax return  $\theta_c \theta_r f'(K) \theta_p \theta_d$ . Instead, the firm can distribute the euro as dividends. The household realises then an after-tax opportunity return  $\theta_p \theta_d r \theta_p$  if the dividends are invested in bonds. This yields the following expression for the cost of capital, which is lower than the interest rate

$$f'(K) = \frac{\theta_p}{\theta_c \theta_r} r < r. \quad (17)$$

### 5.6 Optimal path over time

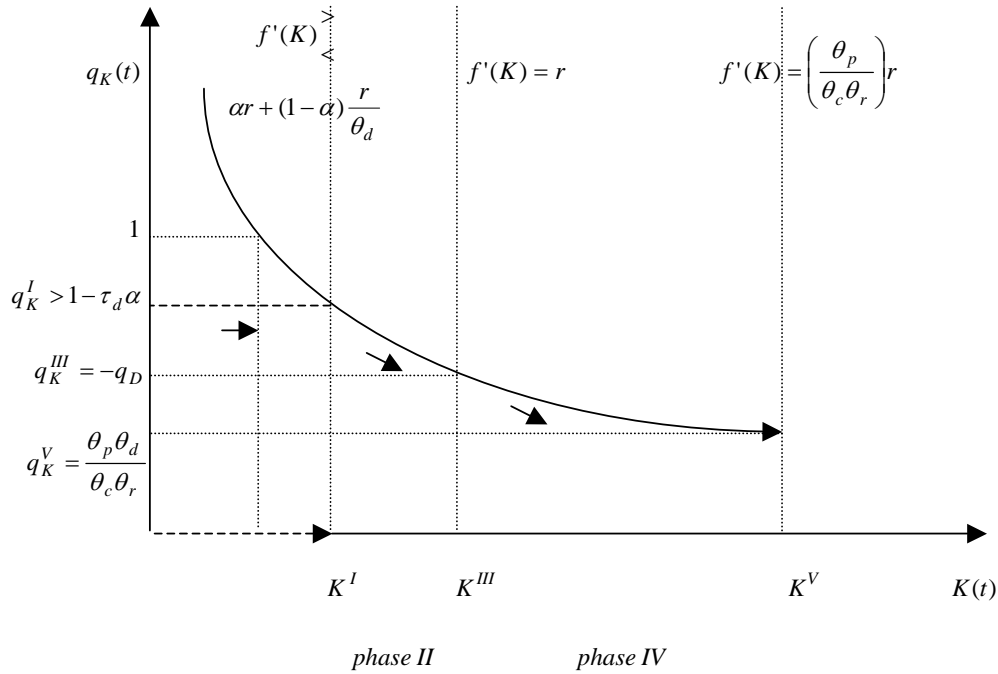
Figure 2 shows how the marginal value of investment  $q_K$  changes with the capital stock over time. The concavity of the production function implies that the marginal productivity of capital and  $q_K$  are decreasing in the capital stock  $K$ . During the initial phase, the firm immediately attracts  $K^I$  units of capital. The marginal value of the firm's capital stock satisfies  $q_K^I > 1 - \tau_d \alpha$ . Afterwards, the firm enters an internal growth phase. Investment is financed with both retained earnings and debt. During this phase, the marginal value of capital exceeds the cost of debt:  $q_K > -q_D$ . At the end of this second phase, the firm has accumulated  $K^{III}$  units of capital. Moreover,  $f'(K) = r$  and  $q_K = -q_D$ . During phase III, the firm redeems its entire debt. During the second internal growth phase, the firm finances investment only with retained earnings. The firm accumulates  $K^V$  units of capital until

$q_K^V = \frac{\theta_p \theta_d}{\theta_c \theta_r} < 1$ . At that point, the firm enters phase V. The firm stops investing and distributes all profits as dividends.

If  $\alpha = 1$ , the firm's initial capital stock is entirely financed with debt so that  $f'(K) = r$ . In fact, the firm jumps immediately towards phase III as the firm does not issue any new equity and does not pass through phase II. The firm does, however, go through phases III, IV and V. In particular, the firm first uses its earnings to redeem its debt, subsequently employs these earnings to finance real investment, and eventually starts distributing the earnings.

If the firm cannot attract debt financing at all (i.e.  $\alpha = 0$ ), it issues new equity during the initial phase until  $q_K(t_1) = 1$ . As discussed in Sinn (1991a), the firm enters an internal growth phase during which investment is financed with retained earnings (phase IV). The mature firm starts distributing dividends when it reaches phase V. In this case, the firm thus does not pass through phases II and III.

figure 2:



The corporate tax on distributed earnings  $\tau_d$  affects the life-cycle of the firm but does not impact the final capital stock (because  $f'(K) = \frac{\theta_p}{\theta_c \theta_r} r$  is not affected). If  $\alpha = 0$ , as derived in Sinn (1991a), an increase in  $\tau_d$  consequently lowers the initial capital stock  $K(t_1)$ . This increases the amount of capital  $K^V - K^I$  that must be accumulated over time. Hence, an



increase in  $\tau_d$  increases the time required by the firm to become mature. The slowing down of capital accumulation, however, becomes less prominent if debt finance is available. Indeed, in the extreme case that the firm can finance the investment entirely with debt, it immediately attract capital such that  $f'(K(t_1)) = r$ . Since this initial capital stock is not affected by  $\tau_d$ , the tax on distributions does not impact the time path for the accumulation of capital at all and the length of the internal growth phase is entirely driven by the difference between  $\theta_c \theta_r$  and  $\theta_p$ .

## 6. Conclusions

In order to investigate the impact of capital income taxation on real investment and its financing, this paper incorporates debt financing in Sinn's dynamic life-cycle model of the firm. The firm's earnings may thus be distributed not only as dividends but also as interest payments. Moreover, they may not only be retained and reinvested but also used to redeem debt. Our analysis determines the firm's optimal source of finance and use of earnings over time, and derives the cost of capital during the entire life cycle of a firm.

If capital gains are taxed less heavily than interest payments are and dividends are taxed at the highest rates, the firm's life cycle consists of five successive phases. Initially, the newly founded firm issues new equity and debt to start its business. Subsequently, the firm enters an internal growth phase during which investment is financed with both debt and retained earnings. When the investment's return has fallen to the level of the interest rate, the firm uses its earnings to redeem its entire debt level rather than for the purpose of real investment. Once the firm has redeemed all its debt, the firm's earnings are used again to finance investment. Eventually the returns on investment drop so much that the firm stops investing and starts distributing its earnings as dividends.

The model could be used to study the effect of share repurchases on the firm's dynamic finance and investment decisions (Brys (2005)). Share repurchases reduce the cost of newly issued equity. Hence, the firm might prefer to forego debt financing entirely. Moreover, an increase in the amount of share repurchases shortens the firm's internal growth phase. The model could allow for unexpected technology shocks. If an unanticipated technology shock enhances the firm's productivity, the firm might want to issue additional new equity.

Another extension involves the introduction of convex adjustment costs as a result of the installation of new capital. In the presence of adjustment costs, the firm's life cycle may consist of two additional feasible phases. First, the firm might pass through an internal growth phase in which it does not use all its earnings for reinvestment but distributes part of its earnings. Investing too many funds in the same period might not be efficient, due to the adjustment costs. Second, after it has been founded, the firm may continue to employ newly issued equity as a source of finance. Intuitively, with adjustment costs raising the costs of large investment flows, the firm may want to spread out issuing new equity over time.

## Appendix A. The preferred sources of finance and the first-order conditions

Given the gross dividends in (2) and a co-state variable  $q_K$  for  $K$  and  $q_D$  for  $D_f$  and Lagrangian multipliers  $\lambda$ ,  $\mu_Q$  and  $\mu_\pi$  for the flow constraints corresponding to, respectively, debt financing, newly issued equity, and dividends, the Lagrangian for maximising the firm's after-tax value is given by

$$L = \left( \frac{1}{\theta_c} + \mu_\pi \right) \left[ \theta_p \theta_d \left[ f(K) - rD_f + \frac{S_f}{\theta_r} + \frac{Q}{\theta_r} - \frac{I}{\theta_r} \right] \right] - (1 - \mu_Q)Q + q_K I + q_D S_f + \lambda [\alpha I - S_f] \quad (18)$$

### A.1. The firm's preferred sources of finance in the steady-state

In comparing the financial instruments available to the firm, we use the change in market value resulting from the substitution of two financial instruments as the evaluation criterion. The Lagrangian (18) is linear in  $S_f$  and  $Q$ . Accordingly, the solutions are boundary solutions. This allows us to ignore the flow constraints in (18), which then reduces to the current-value Hamiltonian  $H$ .

Given  $S_f$  and  $I$ , marginally increasing  $Q$  in (18) measures the marginal advantage of distributing one euro of before-tax retained earnings (RE) and financing investment with an additional euro of newly issued equity (NE) instead:

$$\begin{array}{c} > \\ NE \approx RE \Leftrightarrow \frac{\partial H}{\partial Q} = \frac{\theta_p \theta_d}{\theta_c \theta_r} - 1 = 0 \Leftrightarrow \theta_p \theta_d = \theta_c \theta_r. \\ < \end{array} \quad (19)$$

This yields expression (4) in the main text.

Given values for  $I$  and  $Q$ , marginally raising  $S_f$  in (18) measures the marginal advantage of distributing one euro of before-tax retained earnings (RE) that is replaced by an additional euro of new debt (DF) to finance investment:

$$\begin{array}{c} > \\ DF \approx RE \Leftrightarrow \frac{\partial H}{\partial S_f} = \frac{\theta_p \theta_d}{\theta_c \theta_r} + q_D = 0. \\ < \end{array} \quad (20)$$

The steady-state ( $SS$ ) value of the co-state variable  $q_D = \frac{dM(t^{SS})}{dD_f(t^{SS})}$ , given the value of

$\pi^d(t)$  and because  $\mu_\pi = 0$  (since the firm distributes dividends in the steady state), amounts to (note that tax rates are constant in the steady state):

$$q_D(t^{SS}) = \frac{d M(t^{SS})}{d D_f(t^{SS})} = \int_{t^{SS}}^{\infty} - \left( \frac{1}{\theta_c} + \mu_{\pi} \right) \theta_p \theta_d r(v) \cdot e^{-\int_{t^{SS}}^v \frac{\theta_p}{\theta_c} r(s) ds} dv = \theta_d \left( \int_{t^{SS}}^{\infty} \frac{d \cdot e^{-\int_{t^{SS}}^v \frac{\theta_p}{\theta_c} r(s) ds}}{dv} dv \right) = -\theta_d. \quad (21)$$

Substituting (21) into (20) yields (5) in the main text.

Expression (6) follows from considering substitution of newly issued equity for debt financing:

$$\begin{array}{c} \succ \\ NE \approx DF \Leftrightarrow \frac{dH}{dQ} \Big|_{S_f+Q=CONSTANT} \\ \prec \end{array} = \frac{\partial H}{\partial Q} - \frac{\partial H}{\partial S_f} = -1 - q_D = 0, \quad (22)$$

and substituting (21).

## A.2 The model's first-order conditions

The first-order conditions of the firm's maximisation problem in (3) are:

$$\frac{\partial H}{\partial A} = 0 \quad : \quad q_K = \left( \frac{1}{\theta_c} + \mu_{\pi} \right) \frac{\theta_p \theta_d}{\theta_r} - \lambda \alpha, \quad (23)$$

$$\frac{\partial H}{\partial Q} = 0 \quad : \quad \left( \frac{1}{\theta_c} + \mu_{\pi} \right) \frac{\theta_p \theta_d}{\theta_r} = (1 - \mu_Q), \quad (24)$$

$$\frac{\partial H}{\partial S_f} = 0 \quad : \quad q_D = \lambda - \left( \frac{1}{\theta_c} + \mu_{\pi} \right) \frac{\theta_p \theta_d}{\theta_r}. \quad (25)$$

$$(23) \text{ and } (24) \text{ imply: } q_K = 1 - \mu_Q - \lambda \alpha, \quad (26)$$

$$(24) \text{ and } (25) \text{ imply: } q_D = \lambda + \mu_Q - 1, \quad (27)$$

$$(23) \text{ and } (25) \text{ imply: } q_K = -q_D + \lambda(1 - \alpha). \quad (28)$$

For the costate variables we have:

$$\dot{q}_K - \left( \frac{\theta_p}{\theta_c} r \right) q_K = - \left( \frac{1}{\theta_c} + \mu_{\pi} \right) \theta_p \theta_d f'(K), \quad (29)$$

which, by using (23), can be written as:

$$\dot{q}_K + q_K \left[ \theta_r f'(K) - \frac{\theta_p}{\theta_c} r \right] = -\lambda \alpha \theta_r f'(K). \quad (30)$$

Similarly,

$$\dot{q}_D - \left( \frac{\theta_p}{\theta_c} r \right) q_D = \left( \frac{1}{\theta_c} + \mu_\pi \right) \theta_p \theta_d r, \quad (31)$$

which, by using (25), can be written as:

$$\dot{q}_D + r q_D \left( \frac{\theta_c \theta_r - \theta_p}{\theta_c} \right) = \lambda \theta_r r. \quad (32)$$

Adding (29) and (31) and using (28) yields:

$$\dot{\lambda} = \lambda \frac{\theta_p}{\theta_c} r - \frac{\left( \frac{1}{\theta_c} + \mu_\pi \right) \theta_p \theta_d (f'(K) - r)}{(1 - \alpha)} = \lambda \frac{\theta_p}{\theta_c} r - \frac{\theta_r (q_K + \alpha \lambda) (f'(K) - r)}{(1 - \alpha)}. \quad (33)$$

The firm's initial conditions are  $\frac{\partial \mathcal{M}(t_1)}{\partial K(t_1)} - 1 + \lambda \alpha = 0$ , which implies  $q_K(t_1) = 1 - \lambda \alpha$  (34) as

$\frac{\partial \mathcal{M}(t)}{\partial K(t)} \equiv q_K(t)$  holds by definition, and  $\frac{\partial \mathcal{M}(t_1)}{\partial D_f(t_1)} + 1 - \lambda = 0$ , which implies  $q_D(t_1) = \lambda - 1$  (35)

as  $\frac{\partial \mathcal{M}(t)}{\partial D_f(t)} \equiv q_D(t)$  holds by definition. The transversality constraints are

$$\lim_{v \rightarrow \infty} q_K(t) K(t) e^{-\int_t^v \frac{\theta_p}{\theta_c} r(s) ds} = 0 \quad \text{and} \quad \lim_{v \rightarrow \infty} q_D(t) D_f(t) e^{-\int_t^v \frac{\theta_p}{\theta_c} r(s) ds} = 0 \quad (36).$$

## Appendix B. The model's solution if $\theta_p \geq \theta_p \theta_d \geq \theta_c \theta_r$

$\theta_p \geq \theta_p \theta_d \geq \theta_c \theta_r$  implies that the firm prefers external to internal sources of finance. As a result, the firm directly attracts the optimal amount of capital and starts distributing dividends, which implies that  $\mu_\pi = 0$ . (21) and (35) then imply that  $\lambda = \tau_d$ . Substitution of this result in (34) yields (7).

The value of the co-state variable  $q_K = \frac{d \mathcal{M}(t)}{d K(t)}$  amounts to

$$q_K(t) = \int_t^\infty \left( \frac{1}{\theta_c} + \mu_\pi \right) \theta_p \theta_d f'(K) \cdot e^{-\int_t^v \frac{\theta_p}{\theta_c} r(s) ds} dv. \text{ In view of the constant interest rate and } \mu_\pi = 0,$$

the solution of this integral yields  $\theta_d f'(K) = q_K r$ . Substituting the value of  $q_K$  (from (7)), we then derive the cost of capital of (8).

## Appendix C. The model's solution if $\theta_c\theta_r > \theta_p > \theta_p\theta_d$

Applying the solution procedure of van Loon (1983), this section derives the solution of the firm's problem in (3) if  $\theta_c\theta_r > \theta_p > \theta_p\theta_d$ .

A phase is characterised by the values of  $\pi^d$ ,  $Q$ ,  $I$  and  $S_f$ . Given the first-order conditions and the assumption that  $\theta_c\theta_r > \theta_p > \theta_p\theta_d$ , the first and second step of the solution procedure determines the feasible phases (Appendix C.1) and characterises these phases in terms of the value of the control variables, the state variables, the cost of capital and the co-state variables (Appendix C.2). The third step determines the final phase(s) (Appendix C.3). The chain of feasible phases is obtained if the Lagrange multipliers, the co-state variables and the state variables are proven to be continuous (Appendix C.4). The final step checks whether the first phase of the optimal solution satisfies the initial condition (Appendix C.5).

### C.1. The feasible phases

Phase	$\pi$	$Q$	$I$	$S_f$	
A.	0	0	0	0	infeasible
B.	0	0	0	<0	
C.	0	0	>0	$\alpha I$	
D.	0	0	>0	$0 < S_f < \alpha I$	infeasible
E.	0	0	>0	0	
F.	0	0	>0	<0	infeasible
G.	0	>0	0	0	infeasible
H.	0	>0	0	<0	$\Leftrightarrow \theta_c\theta_r = \theta_p$
I.	0	>0	>0	$\alpha I$	infeasible
J.	0	>0	>0	$0 < S_f < \alpha I$	infeasible
K.	0	>0	>0	0	infeasible
L.	0	>0	>0	<0	infeasible
M.	>0	0	0	0	
N.	>0	0	0	<0	$\Leftrightarrow \theta_c\theta_r = \theta_p$
O.	>0	0	>0	$\alpha I$	$\Leftrightarrow \theta_p \geq \theta_c\theta_r \geq \theta_p\theta_d$
P.	>0	0	>0	$0 < S_f < \alpha I$	infeasible
Q.	>0	0	>0	0	infeasible
R.	>0	0	>0	<0	infeasible
S.	>0	>0	0	0	$\Leftrightarrow \theta_c\theta_r = \theta_p\theta_d$
T.	>0	>0	0	<0	$\Leftrightarrow \theta_c\theta_r = \theta_p = \theta_p\theta_d$
U.	>0	>0	>0	$\alpha I$	$\Leftrightarrow \theta_c\theta_r = \theta_p\theta_d$
V.	>0	>0	>0	$0 < S_f < \alpha I$	infeasible
W.	>0	>0	>0	0	infeasible
X.	>0	>0	>0	<0	infeasible

Phase A. is excluded because, by assumption, it is profitable to invest in at least 1 phase. The Lagrangian (18) is linear in  $S_f$ . This implies that the solutions are boundary solutions. This explains the exclusion of phases D., J., P., and V. Phase G. implies that  $Q = -\theta_r(f(K) - rD_f) < 0$ , which contradicts  $Q \geq 0$  (see (3)).  $\lambda = 0$  in phase F. implies that

$q_K = -q_D$  (see (28)). Taking the time derivative, we establish  $\dot{q}_K = -\dot{q}_D$ . Using (30) and (32), we find that this equation simplifies to  $f'(K) = r$ , which implies that the cost of capital, and therefore the capital stock, are constant during phase F. However, the constraints of the model (see (3)) imply  $\dot{K} = I = \theta_r(f(K) - rD_f) + S_f > 0$ . This contradiction shows that phase F. is not feasible.  $\mu_Q = \lambda = 0$  in Phase K., which implies  $q_K = 1$  (see (26)). This implies that

$\dot{q}_K = 0$  so that (30) simplifies to  $f'(K) = \frac{\theta_p}{\theta_c \theta_r} r$ , which implies that the cost of capital and the

capital stock are constant during phase K. However, the constraints of the model (see (3)) imply  $\dot{K} = I = \theta_r(f(K) - rD_f) + Q > 0$ . This contradiction causes phase K. to be infeasible.

Phase I., L., Q., R., W. and X. are excluded on similar grounds. Phases H., N., S., T. and U. are excluded because they do not satisfy the assumption  $\theta_c \theta_r > \theta_p > \theta_p \theta_d$ . For instance, in

phase U., we have  $\mu_\pi = \mu_Q = 0$ . (24) then shows that this phase is feasible only if  $\theta_p \theta_d = \theta_c \theta_r$ , which contradicts our assumptions. Finally,  $\lambda$  is either 0 or positive in phase O.

If  $\lambda = 0$ , phase O. is not feasible because the cost of capital and the capital stock cannot be constant while the investment level is positive. If  $\lambda > 0$ , (23) implies  $\dot{q}_K = -\alpha \dot{\lambda}$ . Substitution of (29) and (33) then implies that  $f'(K) < r$ . (28) and the values of  $q_K$  and  $q_D$  yield:

$$\lambda(1-\alpha) = \int_t^\infty \left( \frac{1}{\theta_c} + \mu_\pi \right) \theta_p \theta_d (f'(K) - r(v)) \cdot e^{-\int_t^v \frac{\theta_p}{\theta_c} r(s) ds} dv. \text{ Hence, } \lambda > 0 \text{ if and only if } f'(K) \geq r.$$

This contradiction implies that phase O. is not feasible.

## C.2. Characterisation of the feasible phases

*Phase C.* (section 5.2):

$$\pi = 0, \quad Q = 0, \quad I > 0, \quad S_f = \alpha I, \quad \lambda \geq 0, \quad \mu_\pi \geq 0, \quad \mu_Q \geq 0.$$

(2) implies  $I = \frac{\theta_r(f(K) - rD_f)}{(1-\alpha)}$ , and therefore  $S_f = \frac{\alpha}{(1-\alpha)} \theta_r(f(K) - rD_f)$ . A reformulation of

$$(30) \text{ yields } f'(K) = \left[ \frac{q_K}{q_K + \lambda \alpha} \right] \left[ \frac{\theta_p}{\theta_c \theta_r} r - \frac{\dot{q}_K / q_K}{\theta_r} \right], \text{ which, by substitution of (28), implies (14).}$$

*Phase B.* (section 5.3):

$$\pi = 0, \quad Q = 0, \quad I = 0, \quad S_f < 0, \quad \lambda = 0, \quad \mu_\pi > 0, \quad \mu_Q \geq 0.$$

The budget constraint (2) simplifies to  $S_f = -\theta_r(f(K) - rD_f)$ , which implies  $\dot{S}_f = \theta_r r S_f < 0$ .

(30) and  $\lambda = 0$  yield  $f'(K) = \frac{\theta_p}{\theta_c \theta_r} r - \frac{\dot{q}_K / q_K}{\theta_r}$ . The time derivative of  $q_K = -q_D$  (28) implies

$$\frac{\dot{q}_K}{q_K} = \frac{\dot{q}_D}{q_D}. \text{ Solving (32) for } \frac{\dot{q}_D}{q_D} \text{ and substituting it in the cost of capital then yields } f'(K) = r.$$

We now show that  $\mu_\pi > 0$  by contradiction.  $\mu_\pi = 0$  would imply that  $q_K = \frac{\theta_p \theta_d}{\theta_c \theta_r}$  (23) and

therefore that  $\dot{q}_K = 0$ . This would yield  $f'(K) = \frac{\theta_p}{\theta_c \theta_r} r$ , which contradicts the result that

$f'(K) = r$ . Consequently:  $\mu_\pi > 0$ .

(25) can now be written as  $\theta_c \mu_\pi = \frac{\theta_c \theta_r (-q_D) - \theta_p \theta_d}{\theta_p \theta_d}$ . The Lagrange multiplier  $\mu_\pi$  measures

the increase in value of the firm's equity if the firm would redeem an additional unit of debt. Debt redemption is profitable if  $\theta_c \theta_r (-q_D) - \theta_p \theta_d > 0$ . This condition is satisfied because  $q_D < -\theta_d$  (from (21) and  $\mu_\pi > 0$ ) and  $\theta_c \theta_r > \theta_p$ . The firm therefore redeems its entire debt level during phase B.

*Phase E.* (section 5.4):

$$\pi = 0, \quad Q = 0, \quad I > 0, \quad S_f = 0, \quad \lambda = 0, \quad \mu_\pi > 0, \quad \mu_Q \geq 0.$$

(2) implies  $I = \theta_r(f(K) - rD_f)$ . From  $\lambda = 0$  and (30), the cost of capital amounts to

$$f'(K) = \frac{\theta_p}{\theta_c \theta_r} r - \frac{\dot{q}_K / q_K}{\theta_r}.$$

*Phase M.* (section 5.5):

$$\pi > 0, \quad Q = 0, \quad I = 0, \quad S_f = 0, \quad \lambda = 0, \quad \mu_\pi = 0, \quad \mu_Q = \frac{\theta_c \theta_r - \theta_p \theta_d}{\theta_c \theta_r}.$$

(2) implies  $\pi^d = f(K) - rD_f$ .  $\mu_Q = \frac{\theta_c \theta_r - \theta_p \theta_d}{\theta_c \theta_r} > 0$  from (24). Since  $\lambda = 0$  (it can be demonstrated that  $\lambda > 0$  is infeasible; a similar analysis was already presented with respect to phase O. in Appendix C.1), (23) gives rise to  $q_K = \frac{\theta_p \theta_d}{\theta_c \theta_r}$ , which implies that  $\dot{q}_K = 0$ .

Consequently, (30) simplifies to  $f'(K) = \frac{\theta_p}{\theta_c \theta_r} r$ .

### C.3. The final phase

A final phase satisfies the transversality conditions (36), which require that

$$\lim_{t \rightarrow \infty} \left[ \frac{\dot{K}(t)}{K(t)} + \frac{\dot{q}_K(t)}{q_K(t)} - \frac{\theta_p}{\theta_c} r(t) \right] < 0 \text{ and } \lim_{t \rightarrow \infty} \left[ \frac{\dot{D}_f(t)}{D_f(t)} + \frac{\dot{q}_D(t)}{q_D(t)} - \frac{\theta_p}{\theta_c} r(t) \right] < 0. \text{ Phase B. cannot}$$

be the final phase because of the finite debt level (the firm cannot buy back debt forever).

Phase C. is a final phase if

$$\lim_{t \rightarrow \infty} \left( \frac{\theta_r (f(K(t)) - rD_f(t))}{(1 - \alpha)K(t)} \right) < \lim_{t \rightarrow \infty} \left( \theta_r + \frac{\lambda(t)\alpha\theta_r}{q_K(t)} \right) \cdot \lim_{t \rightarrow \infty} (f'(K(t))).$$

Since the concave production function satisfies the Inada condition  $\lim_{t \rightarrow \infty} (f'(K(t))) = 0$ , the condition requires

$$\text{that } \lim_{t \rightarrow \infty} \left( \frac{\theta_r (f(K(t)) - rD_f(t))}{(1 - \alpha)K(t)} \right) < 0 \text{ and therefore that } \lim_{t \rightarrow \infty} (f(K(t))) < \lim_{t \rightarrow \infty} (rD_f(t)).$$

Consequently phase C. cannot be a final phase because it would imply that the firm's revenues decrease below the interest payments. Similar arguments prevent phase E. from being a final phase. Indeed, phase M. is the only feasible final phase because

$$\dot{K}(t) = \dot{q}_K(t) = \dot{D}_f(t) = \dot{q}_D(t) = 0.$$

### C.4. The optimal sequence of phases

This section determines the optimal sequence of phases by analysing the continuity of  $\mu_\pi$ ,  $\mu_Q$ ,  $\lambda$ ,  $q_K$ ,  $q_D$ ,  $K(t)$  and  $D_f(t)$ . Given the final phase M., phase C. may precede phase M.

if  $\mu_\pi(\bar{t}_{CM}) = 0$  and if  $\lambda(\bar{t}_{CM}) = 0$ . This last condition is satisfied if  $\dot{\lambda} \leq 0$  when  $\lambda = 0$  in phase

C. Evaluating (33) in phase C. when  $\lambda = 0$ , we obtain that  $\dot{\lambda} \leq 0 \Leftrightarrow f'(K) \geq r$ . However, since  $\theta_c \theta_r > \theta_p$ , phase M. is characterised by  $f'(K) < r$ . This contradiction implies the discontinuity of  $K(t)$  when going from phase C. to phase M. Consequently, phase C. cannot precede phase M. Phase B. can not precede the final phase either. The cost of capital in

phase B. is  $f'(K) = r$ , while it equals  $f'(K) = \frac{\theta_p}{\theta_c \theta_r} r$  during phase M. Because  $\theta_c \theta_r > \theta_p$ ,

this would again imply a discontinuous jump in  $K(t)$ . Phase E. precedes phase M. if

$\mu_\pi(\bar{t}_{EM}) = 0$ , which is satisfied if  $\dot{\mu}_\pi \leq 0$  when  $\mu_\pi = 0$  during phase E. Taking the time

derivative of  $q_K = \left( \frac{1}{\theta_c} + \mu_\pi \right) \frac{\theta_p \theta_d}{\theta_r}$ , and using (30) and the fact that  $\lambda = 0$  during phase E.,

we obtain  $\dot{\mu}_\pi = \left( \frac{1}{\theta_c} + \mu_\pi \right) \left[ \frac{\theta_p}{\theta_c} r - \theta_r f'(K) \right]$ . Consequently,  $\dot{\mu}_\pi \leq 0$  when  $\mu_\pi = 0$  in phase E.

if  $f'(K) \geq \frac{\theta_p}{\theta_c \theta_r} r$  which holds along the entire path. It implies that Phase E. precedes phase

M. We now demonstrate that the firm after phase C. passes through phase B. before it



enters phase E. Comparing the cost of capital in phase C. and E. shows that the continuity of the cost of capital and the capital stock is guaranteed if  $\lim_{C \rightarrow E} \left[ \frac{q_K}{q_K + \lambda \alpha} \right] = 1$ . This condition is equivalent with  $\lim_{C \rightarrow E} \lambda = 0$ . Hence, we have to prove the continuity of  $\lambda$ , which holds if  $\dot{\lambda} \leq 0$  if  $\lambda = 0$  in phase C. Evaluating (33) at  $\lambda = 0$ , we obtain that  $\dot{\lambda} \leq 0 \Leftrightarrow f'(K) \geq r$ . Therefore, the continuity of  $\lambda$  can be guaranteed only if the firm, after phase C., passes through phase B., where  $f'(K) = r$ , before it enters phase E.

### C.5. The initial condition

This section checks whether the optimal path satisfies the initial condition. (30) implies that  $\dot{q}_K \leq 0$  along the optimal path as long as  $f'(K) \geq \frac{\theta_p}{\theta_c \theta_r} r$ . A sufficient condition for the initial condition (34) to be satisfied is that  $\dot{q}_K \leq 0$  when  $q_K(t_1) = 1 - \lambda \alpha$ . From (30), we obtain that  $\dot{q}_K \leq 0$ , evaluated at  $q_K(t_1) = 1 - \lambda \alpha$  and therefore at the investment level  $K(t_1)$ , as long as  $f'(K(t_1)) \geq (1 - \lambda \alpha) \frac{\theta_p}{\theta_c \theta_r} r$ . Since the initial capital stock is determined endogenously, this condition is a requirement on the shape of the production function. If it is satisfied, the firm will issue the optimal initial amount of new equity and debt and will jump towards the optimal path. From (32), we observe that  $\dot{q}_D = \lambda \theta_r r - r q_D \left( \frac{\theta_c \theta_r - \theta_p}{\theta_c} \right) \geq 0$  along the optimal path if  $\theta_c \theta_r > \theta_p$ . Evaluating (32) at  $q_D(t_1) = \lambda - 1$ , it can be demonstrated that  $\dot{q}_D \geq 0$ , which implies that the second initial condition is satisfied as well. Because  $\lambda > 0$  in the initial condition, the firm jumps towards phase C. and not towards phases B., E. or M. (phase C. it is the only phase where  $\lambda > 0$ ).

## Appendix D. The initial phase if $\theta_c \theta_r > \theta_p > \theta_p \theta_d$

### D.1. The initial period's co-state variables

If  $\theta_c \theta_r > \theta_p > \theta_p \theta_d$ , profitable investment opportunities remain after the initial period's investment. As a result, the firm does not directly start distributing dividends, which implies that  $\mu_\pi(t_1) > 0$ . The initial period's value of the co-state variable then equals

$$q_D(t_1) = \int_{t_1}^{\infty} - \left( \frac{1}{\theta_c} + \mu_\pi \right) \theta_p \theta_d r(v) \cdot e^{-\int_{t_1}^v \frac{\theta_p}{\theta_c} r(s) ds} dv < -\theta_d. \text{ The initial condition (35) then implies}$$

that  $\lambda(t_1) < \tau_d$ . Substitution of this result in (34) then implies (9).

### D.2. The initial period's cost of capital

- $\alpha = 1$

If  $\alpha = 1$ , the firm finances the investment entirely with debt. Since  $\lambda$  is the gain of an additional unit of debt, the firm stops to issue debt when  $\lambda = 0$ . Appendix C.1. derived that

$$\lambda(1-\alpha) = \int_t^\infty \left( \frac{1}{\theta_c} + \mu_\pi \right) \theta_p \theta_d (f'(K) - r(v)) \cdot e^{-\int_t^v \frac{\theta_p}{\theta_c} r(s) ds} dv. \quad \text{Under the assumption of a concave}$$

production function and a constant interest rate, the firm then continues to finance investment with debt until  $f'(K) = r$ .

- $\alpha = 0$

If  $\alpha = 0$ , this section derives a condition under which the cost of capital of initial investment financed with newly issued equity is  $f'(K(t_1)) > \frac{r}{\theta_d}$ .

By definition, the marginal value of capital and the cost of capital are equal to:

$$q_K = q_K(K_2') = \frac{\theta_p \theta_d}{\theta_c \theta_r} \quad \text{and} \quad f'(K) = \frac{\theta_p}{\theta_c \theta_r} r \quad \text{if} \quad K = K_2', \quad (37)$$

$$q_K = q_K(K_1') = 1 \quad \text{if} \quad K = K_1'. \quad (38)$$

Moreover, if  $\alpha = 0$ , (30) and  $\dot{K} = \theta_r f(K)$  imply that during the internal growth phase:

$$\frac{dq_K}{dK} = \frac{q_K}{f(K)} \left[ \frac{\theta_p}{\theta_c \theta_r} r - f'(K) \right] \quad \text{for} \quad K_1' \leq K \leq K_2'. \quad (39)$$

We define the function  $s(K) = \frac{\theta_d f'(K)}{r}$ . By definition, it holds that:

$$s = s(K_1) = 1 \quad \text{and} \quad f'(K) = \frac{r}{\theta_d} \quad \text{if} \quad K = K_1, \quad (40)$$

$$s = s(K_2') = \frac{\theta_p \theta_d}{\theta_c \theta_r} \quad \text{and} \quad f'(K) = \frac{\theta_p}{\theta_c \theta_r} r \quad \text{if} \quad K = K_2'. \quad (41)$$

The remainder determines when  $q_K(K_1) < 1$ . The capital stock  $K_1$  is characterised by  $f'(K_1) = \frac{r}{\theta_d}$ . If  $q_K(K_1) < 1$  and given that  $q_K(K_1') = 1$ , the initial period's capital stock  $K_1'$  is characterised by  $f'(K_1') > \frac{r}{\theta_d}$  (under the assumption of a concave production function and

given that  $\frac{dq_K}{dK} \leq 0$ ). In view of  $-\infty < dq_K(K)/dK < 0$  for  $0 < K < K_2'$ , (37), (38), (40) and (41), a sufficient condition for  $q_K(K_1) < 1$  is  $\frac{\partial q_K(K)/\partial K}{q_K(K)} > \frac{\partial s(K)/\partial K}{s(K)}$ . Hence,

$$\frac{\partial q_K(K)/\partial K}{q_K(K)} > \frac{\partial s(K)/\partial K}{s(K)} \Leftrightarrow \frac{1}{f(K)} \left[ \frac{\theta_p}{\theta_c \theta_r} r - f'(K) \right] > \frac{f''(K)}{f'(K)} \Leftrightarrow 1 - \frac{\frac{\theta_p}{\theta_c \theta_r} r}{f'(K)} < \frac{(1-\beta)/\beta}{\sigma}, \quad (42)$$

where  $1-\beta = \frac{f(K) - Kf'(K)}{f(K)}$ ,  $\beta = \frac{Kf'(K)}{f(K)}$ , and  $\sigma = -\frac{\left[ \frac{f(K) - Kf'(K)}{Kf'(K)} \right]}{\left[ \frac{f''(K)f(K)}{f'(K)f'(K)} \right]}$ .

(42) and  $f'(K) < \frac{r}{\theta_d}$  for  $K > K_1$  imply that a sufficient condition for  $q_K(K_1) < 1$  is

$$\left[ 1 - \frac{\theta_p \theta_d}{\theta_c \theta_r} \right] < \frac{(1-\beta)/\beta}{\sigma}. \quad (43)$$

- $0 \leq \alpha < 1$

$\alpha = 1$  implies that  $f'(K(t_1)) = r$ .  $\alpha = 0$  implies that  $f'(K(t_1)) > \frac{r}{\theta_d}$  if (43) is fulfilled. Since the investment is  $(\alpha \cdot 100)\%$  financed with debt and  $((1-\alpha) \cdot 100)\%$  with newly issued equity, it follows that  $f'(K(t_1)) > \alpha r + (1-\alpha) \frac{r}{\theta_d} \Leftrightarrow \left[ 1 - \frac{\theta_p \theta_d}{\theta_c \theta_r} \right] < \frac{(1-\beta)/\beta}{\sigma}$ . (12) follows if  $\sigma = 1$ .

## Appendix E. The length of the internal growth phase

We assume that  $f(K) = AK^\beta$ . In view of  $f'(K^{ss}) = \frac{\theta_p}{\theta_c \theta_r} r$  ((17)) and under the assumption that the initial period's capital stock satisfies  $f'(K^I) = \frac{r}{\theta_d}$ , the steady-state capital stock and

the initial period's capital stock amount to  $K^{ss} = \left[ \frac{\theta_p r / \theta_c \theta_r}{A\beta} \right]^{\frac{1}{\beta-1}}$  and  $K^I = \left[ \frac{r / \theta_d}{A\beta} \right]^{\frac{1}{\beta-1}}$ . Notice

that  $\frac{dK}{dt} = \theta_r \cdot A \cdot K(t)^\beta$  if  $\alpha = 0$ . Using  $K^I$  as the initial condition, we find that the solution of

the differential equation is  $K(t) = \left[ \frac{A\beta}{r/\theta_d} + (1-\beta)\theta_r A \cdot t \right]^{1/(1-\beta)}$ . In view of  $K^{SS} = K(t^{SS})$ , this

expression for the capital stock implies that  $t^{SS} = \frac{\beta}{(1-\beta) \cdot \theta_r \cdot r} \left[ \frac{\theta_c \theta_r}{\theta_p} - \theta_d \right]$ . If  $\theta_c \theta_r > \theta_p \theta_d$ ,

we then obtain that  $\frac{\partial t^{SS}}{\partial \beta} > 0$ . Hence, the length of the internal growth phase is increasing in  $\beta$ .

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